Estimation of Coherence Region Shapes in Polarimetric SAR Interferometry

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Synthetic aperture radar (SAR) is a well-established technology for terrain imaging. Its capability for all-weather operation and range-independent resolution make it effective in situations where other imaging technologies cannot be applied. Its coherent nature makes interferometric observations possible, for applications such as terrain elevation mapping and ocean-current measurement. Conventional interferometric SAR (INSAR) operates at a single frequency and polarization. INSAR data from terrain are interpreted using models that assume the terrain to be a scattering surface; volume scattering is treated as a source of decorrelation, causing degradation of the interferogram. A current focus of research in INSAR is its enhancement to allow characterization of volumetric scattering processes, making possible such applications as estimation of forest canopy height, mapping of bare-earth elevation, and estimation of biomass density. This requires the use of multiple frequencies and/or polarizations to make independent observations of the scattering processes in each IFSAR pixel.

A promising technique for volumetric SAR imaging is polarimetric SAR interferometry (POLINSAR), in which the full polarimetric scattering matrix is collected over both apertures of an INSAR pair. After calibration, each image pixel contains three independent polarimetric channels (for example, HH, HV and VV). In a generalization of standard INSAR processing, the images are registered and the cross-products of all pairs of channels are averaged over small regions. The result is a 6x6 covariance matrix for each multilook pixel. The key challenge of POLINSAR processing is to interpret this matrix as the result of superposed, polarization-dependent scattering mechanisms, and from this to estimate parameters such as canopy height, attenuation coefficient and bare-earth interferometric phase.

Current methods of POLINSAR interpretation rely on a generalization of the complex interferometric coherence to polarimetric measurements. For any polarization vector (coherent combination of polarizations), coherence function can be computed from the 6x6 POLINSAR covariance as a ratio of quadratic forms, as described below. Thus the POLINSAR data define a polarization-dependent coherence function for each multilook pixel; the set of possible coherences is called the *coherence region*. The volumetric scattering models of Cloude-Papathanassiou [1] and Treuhaft-Siquiera [2] predict that, in the absence of noise, decorrelation and sampling error, the coherence region is a straight line segment whose location and orientation depend on the canopy height, attenuation and bare-earth height. Based on these models, terrain characterization becomes a problem of fitting a line segment to the coherence region and inverting the model function to extract the parameters.

The extraction of the shape of the coherence region from the POLINSAR data is a challenging problem. For computational efficiency, the region must be represented by a small number of samples, yet the scattering models rely on an accurate fit of a straight line to the region. It must also be possible to detect deviations from the linear shape predicted by the models. Cloude and Papathanassiou [1] propose the use of the stationary points of the complex coherence as a sample set, and interpret this set as a decomposition into independent scattering mechanisms. Tabb *et al.*

[3] present an algorithm for computation of the coherences with maximum and minimum phase, and demonstrate that in most cases a line-segment coherence region is better characterized by these points than by the stationary points of coherence. In the general case, it is questionable whether the coherence region can be characterized adequately with so few samples. The problem calls for an algorithm that finds a sample set of reasonable size that characterizes the shape of the coherence region in all cases.

We now present an algorithm that computes samples of an approximation to the outer boundary of the coherence region to any specified density. The polarizations are restricted to be baseline-copolar (same polarization in both apertures); the recent work of van Zyl and Kim [4] has shown that this constraint preserves the useful information while suppressing undesired effects of the polarimetric phase. Following [1], the 6x6 POLINSAR covariance matrix can be separated into 3x3 matrices:

$$R = \begin{bmatrix} T_{11} & \Omega_{12} \\ \Omega_{12}^H & T_{22} \end{bmatrix} \tag{1.1}$$

where T_{11} and T_{22} are the polarimetric covariance matrices for apertures 1 and 2 respectively, and Ω_{12} is the polarimetric cross-covariance matrix between the apertures. The polarization is specified by a three-element complex unit vector w. The baseline-copolar coherence function is

$$g\left(w\right) = \frac{w^H \Omega_{12} w}{\sqrt{\left(w^H T_{11} w\right)\left(w^H T_{22} w\right)}}. \tag{1.2}$$

For purposes of computation, we introduce the modified coherence

$$\bar{g}(w) = \frac{w^H \Omega_{12} w}{w^H T w} \tag{1.3}$$

where $T = (T_{11} + T_{22})/2$. We have that $\arg \bar{g}(w) = \arg g(w)$ and $\left|\bar{g}(w)\right| \le \left|g(w)\right|$, so the interpretation of $\bar{g}(w)$ as a coherence is reasonable. It can be divided into its real and imaginary parts using the decomposition

$$\Omega_{12} = A + jB, \tag{1.4}$$

where the matrices

$$A = \frac{1}{2}\left(\Omega_{12} + \Omega_{12}{}^H\right) \tag{1.5}$$

and

$$B = \frac{1}{2j}\left(\Omega_{12} - \Omega_{12}{}^H\right) \tag{1.6}$$

are Hermitian. Consider the problem of finding the minimum and maximum of $\mathrm{Re}\,\bar{g} = \left(w^H A w\right) / \left(w^H T w\right)$. Because $\bar{g}$ is a continuous function of w, the coherences that attain these extrema lie on the boundary of the coherence region. By a standard result (page 39 of [5]), the extremizing vectors w are solutions to the generalized eigenvalue problem

$$Aw = l\,Tw\,. \tag{1.7}$$

Each l is the value of $\mathrm{Re}\,\bar{g}$ for the corresponding w; this allows the desired solutions to be identified, yielding two points on the boundary.

Other boundary points can be computed using phase rotations. We can write

$$\bar{g}\,e^{jf} = \frac{w^H\left(\Omega_{12}e^{jf}\right)w}{w^H Tw} \tag{1.8}$$

and make the decomposition

$$\Omega_{12}e^{jf} = \tilde{A} + j\tilde{B} \tag{1.9}$$

where

$$\tilde{A} = A\cos f - B\sin f \tag{1.10}$$

and

$$\tilde{B} = A\sin f + B\cos f \tag{1.11}$$

are Hermitian matrices. The minimum and maximum of $\mathrm{Re}\,\bar{g}\,e^{jf}$ can be found by solving the generalized eigenvalue problem

$$\tilde{A}w = l\,Tw \tag{1.12}$$

and evaluating Equation (1.8) at the resulting eigenvectors. This yields two points on the boundary of the region of possible values of $\bar{g}\,e^{jf}$. Multiplying them by e^{-jf} yields two points on the boundary of the modified coherence region. Every angle in the interval $[0,p)$ produces a different pair of boundary points, except at sharp corners. Thus the boundary can be determined to any desired degree of accuracy by solving the eigenvalue problem for a sufficiently dense sampling of rotation angles. The boundary of the standard coherence region is found approximately by evaluating Equation (1.2) at these solutions, so that all of the approximate boundary points lie on or inside the true boundary.

This procedure is guaranteed to trace out the entire boundary of the modified coherence region if and only if the region is convex. The mathematical literature assures us that this is the case. To see this, define

$$v = T^{1/2}w \tag{1.13}$$

so that Equation (1.3) becomes

$$\bar{g} = \frac{v^H T^{-1/2}\Omega T^{-1/2}v}{v^H v}\,. \tag{1.14}$$

Here $T^{1/2}$ is the positive definite Hermitian square root of T. So the modified coherence region is the set of possible values of the quadratic form $v^H T^{-1/2}\Omega T^{-1/2}v$ under the constraint $v^H v = 1$. This set is called the *field of values* of the square matrix $T^{-1/2}\Omega T^{-1/2}$. Its properties are presented in chapter 1 of [6]. In particular, it is proven that the field of values is a convex subset of the complex plane for any matrix. The standard coherence region is not guaranteed to be exactly convex, but it is very nearly so if $T_{11} \approx T_{22}$.

We demonstrate the algorithm by presenting plots of the coherence region for selected locations in a SIR-C image pair collected over an area near Lake Baikal (the same pair used for the examples in [1]). The covariance data for Figure 1 are from a riverbank, for Figure 2 from a field with strong backscatter. The light blue points mark the baseline-copolar coherences computed on a large number of polarizations. For comparison, the stationary coherences, without the baseline-copolar constraint ("magnitude diversity") are marked with yellow dots, and the phase extrema ("phase diversity") with red dots. The approximate boundary is marked with a red line. The rotation angle f is stepped in increments of 3 degrees, yielding 120 boundary points from 60 generalized eigenvalue problems.

The boundary sampling is very effective in revealing the shape of the coherence region, even in cases where the brute-force sampling becomes sparse because of rapid change of coherence with polarization. The extrema of magnitude and phase are located on the boundary, so boundary sampling determines them to within an error due to the finite sample density. Deviations of the coherence region from the ideal straight-line shape, in particular the triangular shape in Figure 1, are made clear by the boundary sampling.

The boundary computation algorithm is a significant new technique for interpretation of polarimetric IFSAR data. It can determine the shape of the coherence region reliably, in greater detail than earlier methods, at reasonable computational cost. As a source of inputs to vegetation scattering models, it will enhance the accuracy of the estimation of vegetation characteristics from POLINSAR data.

References:

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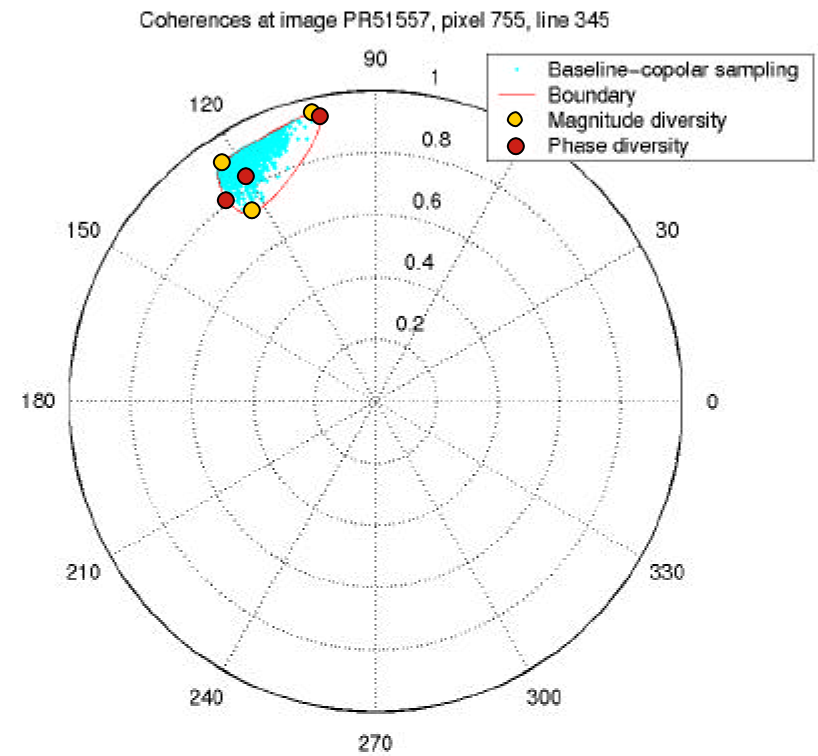

Figure 1. Coherence region on a riverbank.

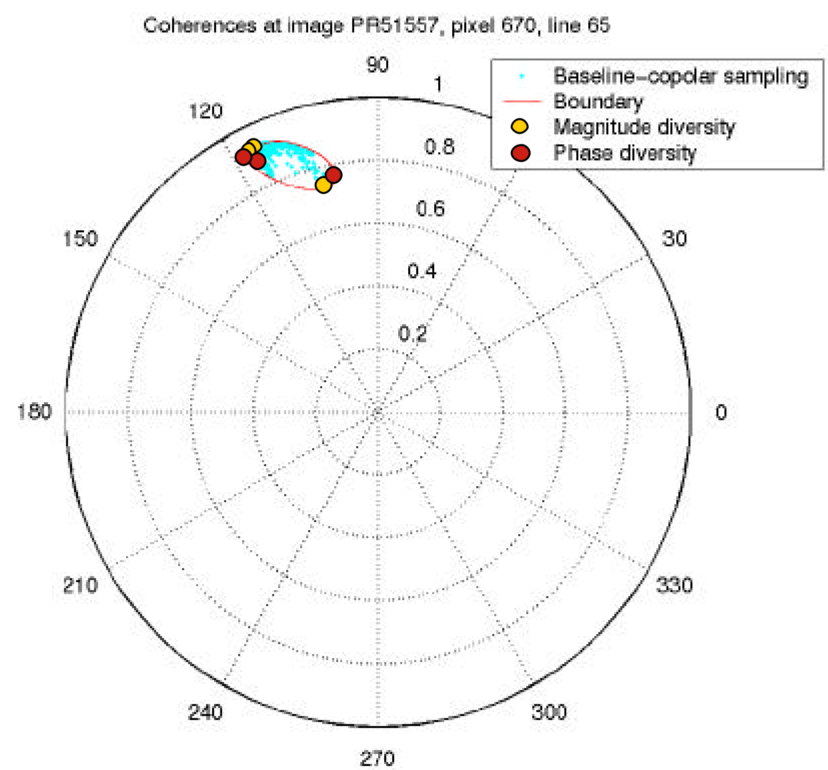

Figure 2. Coherence region on a field with strong scattering.